The distinct element method: a new way to study the behaviour of ancient masonry structures under static or dynamic loading, by the use of numerical modelling

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THE DISTINCT ELEMENT METHOD : A NEW WAY TO STUDY THE BEHAVIOUR OF ANCIENT MASONRY STRUCTURES UNDER STATIC OR DYNAMIC LOADING BY THE USE OF NUMERICAL MODELLING

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SUMMARY

Numerical modelling methods are regularly used for the design of new buildings and sometimes, for the stability analysis of ancient buildings. But ancient buildings or monuments are much different from new constructions because of their blocky (stone) structure. This is why the Finite Element Method or any continuous modelling approach may be not suitable for their analysis. To represent the blocky structure of ancient monuments, we used the Distinct Element Method which allows the representation of all joints and blocks and is able to simulate their non-linear behaviour very easily. Furthermore, the method allows dynamic simulations in a very physical way, easy to understand. Consequently, we modelled the behaviour of a stone wall under different types of seismic loadings to demonstrate the efficiency of the method. The dynamic calculations we have run helped us to understand the status of conservation of similar existing structures.

INTRODUCTION

Numerical modelling methods have reached a very high level of development during the last few years and gained a remarkable success in many fields. Recently, the Distinct Element Method (DEM) became one of the best techniques used for the analysis of fractured rock masses. Many attempts to study the stability of historical monuments such as tombs, pylons and stone colossi, under static loads have been developed (Verdel et al 1993). Lemos (1987) and Hart, Jing & Stephansson (1990) introduced the DEM in the field of dynamic modelling and applied it to study the dynamic behaviour of dams subjected to seismic loads as well as blasting in mines. In that field, the DEM is now also used in soil dynamics, specially for the analysis of non cohesive soils (represented by particles assemblages) under dynamic excitation as proved by the number of papers presented at the 10th International Conference on Earthquake Engineering, Madrid, 1993.

Usually, the main difficulty in modelling an old masonry building is to take into account the non-linear behaviour of the masonry. Furthermore, the models dealing with this type of structures presently do not consider correctly the presence of joints (mortar) between masonry blocks or cracks through the masonry. The main advantage of the DEM is to propose a representation of these discontinuities which has a physical sense.

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In this paper, the authors are trying to emphasize the usefulness of the DEM for the dynamic analysis of ancient masonry structures such as historical monuments. The main principles of the DEM are firstly exposed. Then, in order to quantify the influence of the mechanical characteristics of stones and mortars on the seismic response of an historical masonry building, nine models of a simple masonry wall have been tested. Each of them represents a particular situation of the degradation of mortars and stones.

PRINCIPLES OF THE DISTINCT ELEMENT METHOD

General principles.

The DEM, firstly proposed by Cundall (1971), has been developed to study jointed media subjected to quasi-static or dynamic loads. The medium studied by this method is simulated as an assemblage of discrete blocks (rigid or deformable) which interact through corners and edge contacts (figure 1). Discontinuities are regarded as boundary interactions between these blocks and the method utilizes an explicit time-stepping (dynamic) algorithm which allows large displacements, large rotations and general non-linear constitutive behaviour for both the matrix and the discontinuities. Furthermore, a complete detachment of blocks is possible and new contacts are automatically recognized.

The fundamental equations of the DEM.

The numerical formulation of the DEM was coded in a computer program called UDEC (Universal Distinct Element Code), which takes into account the blocky system geometry, by considering polyhedral blocks. The fundamental equations for the establishment of the calculations by this method are :

- the force-displacement law which relates the forces developed at block contacts (or at gridpoints of deformable blocks) to the relative displacements. This constitutive relation may be linear-elastic or elastic-plastic under Coulomb failure criterion control ;

- the motion equation which defines the motion of the blocks. This equation allows the new kinetic quantities (accelerations, velocities and displacements of grid points) to be determined.

The solution procedure is explicit in the time domain. The two basics sets of calculations corresponding to the execution of the calculation scheme are illustrated on figure 2.

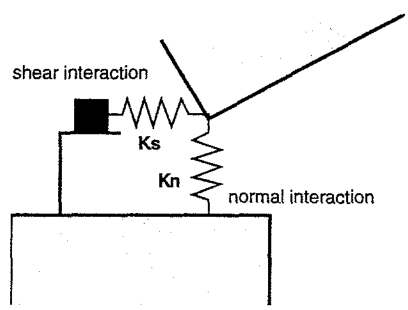

Figure 1. Modelling of the interaction between two discrete blocks in the DEM.

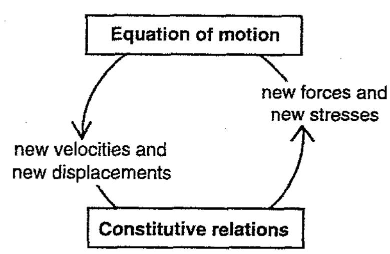

Figure 2. Time marching scheme for solving equations in DEM.

Dynamic modelling by DEM.

1. Representation of the damping.

The energy losses in a natural system modelled with UDEC can take two forms : in the case of quasi-static problems, an adaptive viscous (mass proportional) damping is used and automatically adjusted by the code to accelerate calculations ; in the case of dynamic problems, damping is chosen more realistic and may be given a mass proportional and/or a stiffness proportional component. Such a two components damping is called the Rayleigh damping. The mass proportional damping has an effect similar to that of a block system drawn in a viscous fluid. The stiffness proportional damping is physically equivalent to that produced by dashpots across contacts which damp the relative motion of blocks.

2. Representation of boundary conditions (modelling infinite and semi-infinite bodies).

In static analysis, fixed boundaries can be realistically placed at some distance from the region of interest. In dynamic problems, however, such boundary conditions cause the reflection of outward propagating waves back into the model and do not allow the necessary energy dissipation. The use of a larger model can minimize the problem, since material damping will absorb most of the energy of reflected waves, but such solution is computer-time consuming (Itasca,1991). In UDEC, two solutions are proposed to avoid this problem :

- viscous (non-reflecting) boundaries using the formulation of Lysmer & Kuhlmeyer (1969),
- free-field boundaries may be introduced in seismic analysis of surface and embedded structures where the seismic input is normally represented by a plane wave propagating upwards. In such case, the lateral boundary of the model must account for the free-field motion at those boundaries. The goal is to "enforce" the free-field motion in such a way that lateral boundaries retain their non-reflecting properties. In UDEC, this procedure involves the execution of one-dimensional free-field calculations in parallel with the blocky system analysis. Furthermore, the lateral boundaries are coupled to the free-field grid by the mean of those viscous dashpots.

3. Simulation of dynamic loadings.

A dynamic loading in UDEC can be applied as a velocity, force or stress time history. For simulating an earthquake with the code UDEC, there are two ways :

- introducing the recorded signal,
- applying an harmonic loading history which can model either P-wave, S-wave or a combination of both.

4. Time-step optimization.

The central difference scheme, used in UDEC to integrate the equations of motion, requires the use of a time-step (or time interval) which has to be chosen carefully because of the physical significance of time in dynamic analysis. UDEC provides a time-step optimization depending on the stiffness of blocks and joints, and the size of zones (in case of deformable blocks). For most cases, this optimization insures the numerical stability of calculations.

THE MAIN ADVANTAGES OF THE DEM IN MASONRY MODELLING

The main advantages of using the DEM in modelling historical structures are :

- the representation of contacts, by the mean of a normal and shear stiffness, the behaviour of which being controlled by available criteria (Mohr-Coulomb, tension cut-off, etc.), makes possible to model the assembly of stones bonded by mortars in a realistic manner ;
- the soil-structure interaction can be modelled in a physical way ;
- the degradation of stones and mortars can be easily modelled by changing their mechanical characteristics ;
- the method gives both static and dynamic solutions ;
- it is possible to study some reinforcement schemes by introducing blots elements or structural supporting system in the model ;
- the method was firstly set up for 2-D problems analysis, but it has also been developed for 3-D problem analysis (Cundall, 1988) in order to describe more complicated geometries.

MASONRY STRUCTURES AND EARTHQUAKES

The observation of earthquake damages into masonry structures shows that there are three main factors affecting the earthquake performance of masonry buildings : the status or quality of mortars, the quality of workmanship, and the direction of the seismic waves. For the first two parameters, the modified version of the Mercali Intensity Scale proposed by Richar (1958), classified masonry buildings into four categories, as shown on table 1.

In most cases, one or two diagonal cracks directions are also observed on masonry walls which have been subjected to earthquakes. Accordingly, the shear strength is critical for this type of buildings, as well as the cohesion and the tensile strength of mortars which play the role of bonding the masonry blocks together and may control sliding phenomena.

Category	Workmanship	Mortar	Reinforced	Designed to resiste lateral forces
A	Good	Good	Yes	Yes
B	Good	Good	Yes	No
C	Ordinary	Ordinary	No	No
D	Bad	Poor	No	No

Table 1. Classification of masonry buildings, Newmark (1971).

THE MODELLING OF MASONRIES

During the last few years, extensive research works have been carried out on the analysis of the seismic response of masonry buildings (see Clough 1975, Tomazevic 1987, Gentil 1991). In particular, much effort has been devoted to the definition of simplified structural models capable of representing the behaviour of masonry structures. The major part of this research was realized by either equivalent continuous or idealized (homogenized) models. On another hand, the recent tentatives for modelling the real behaviour of masonries as an anisotropic and no tension material made by Pietruszczak 1991), have undergone many difficulties, especially in regard with the non-linearity.

Otherwise, because of the discontinuous nature of its models, the DEM offers a good opportunity to analyse the behaviour of blocky systems, where mortar/blocks and blocks/blocks interactions are simulated by spring slider systems which allow the evaluation of shear and normal forces at contacts.

Case study 1 : a masonry wall.

We firstly tried to demonstrate the capacity of the DEM to model different masonry wall failure mechanisms according to the mechanical characteristics of mortars, and the type of seismic waves. A simple masonry wall subjected to dynamic loading in its plane wall modelled.

1. Construction of the models and model materials.

The geometry of the model represents a simple masonry wall which is 10 m high and 30 m long (figure 3). All the calculations are carried out in plane stress conditions, and the loading is applied at the base of the wall.

As we know, the values of the elastic and shear moduli influence the dynamic properties of the masonry building. So, the choice of these moduli is very important to determine the response of structures which are known as heterogeneous anisotropic media. The load-bearing capacity tests realized by Sheppard et al (1986), for different types of old masonry walls, including the walls in its original and strengthened state, indicate a wide range of expected values of mechanical properties.

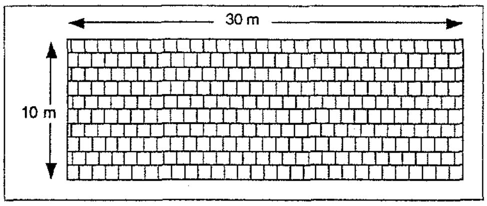

Figure 3. The geometry of the model.

To accomplish our models in a realistic manner and because of the lack of experiments carried out on masonry buildings until now, it was decided to take the characteristics of the model tested by Tomazovic (1992) as a reference. All chosen mechanical parameters are given in table 2.

Mechanical properties			
Blocks	Density	1821	kg/m3
	Young's modulus	20	MPa
	Poisson's ratio	0,35	
	Bulk modulus	2000	MPa
	Shear modulus	820	MPa
Mortar	Normal stiffness	20000	MPa
	Shear stiffness	8200	MPa
	Friction angle	35	Degree
	Cohesion	10000 - 0	MPa
	Tensile strength	10000 - 0	MPa
Loading and dynamic properties			
Stress amplitude		2	MPa
Frequency		10	Hz
Duration (chosen for practical reasons)		2	s
Critical Damping ratio		10 %	
Fundamental frequency (calculated)		30	Hz

Table 2. Mechanical and dynamical properties.

2. Modal Analysis.

To test the capacity of the code UDEC to estimate the fundamental frequency and the modes of vibration of the model, a modal analysis was realized separately using a finite element code. The fundamental frequency obtained in this analysis corresponds perfectly to the one calculated by UDEC (estimated by reproducing undamped oscillations under sudden gravity loading). Furthermore, the shape of the fundamental modes of vibration of the model, obtained by the FEM, are shown on the figure 4. As can be seen, the calculated fundamental mode of vibration reflects a monolithic behaviour of the wall.

It must be noted that the modal analysis realized by the FEM didn't take into consideration the presence of joints and was carried out on a continuous model. This is why the analysis carried out with UDEC, have been realized with fictitious joints (joints having the same mechanical characteristics as the blocks).

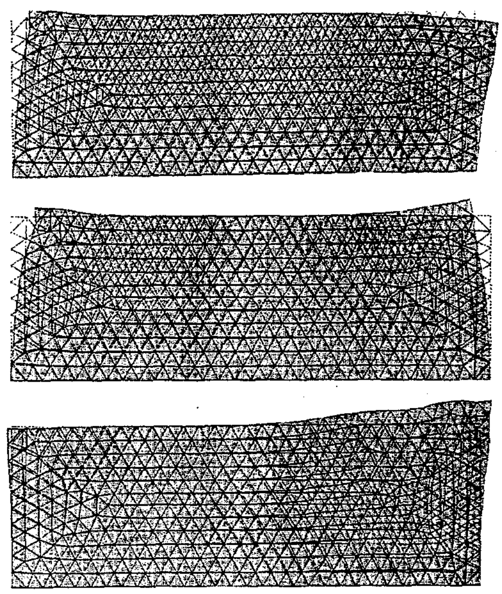

Figure 4. The first three mode shapes of the model detremined by a finite elemnt modal analysis.

3. Modelling by UDEC : calculation sequence.

The stress distribution as well as the state of the structure before the application of the seismic action influence the propagation of the seismic waves across the structure. Therefore, a preliminary stage of calculation is necessary to calculate the initial stress distribution in the model. This stage consists of a quasi-static gravity loading and uses an adaptive automatic damping. It is followed by the dynamic loading.

The dynamic (or earthquake) loading consists of a harmonic wave propagating upward and applied at the base of the model. For this stage, viscous boundaries have been introduced at the base of the model to prevent the reflected wave to return upward in the model.

Two modes of loading have been considered : a shear wave only and a combination of both. Amplitude, frequency and duration of the input loading are indicated on table (2).

To proceed a realistic physical damping, Rayleigh damping was used and the critical damping ratio was given the value of 10 %.

4. Damage propagation and failure mechanisms

The response of the model under the different types of dynamic loading is shown on figures 5 and 6. Each figure illustrates the deformation of the wall during the dynamic loading, at different time intervals. The deformation of blocks on figure 5 has been magnified to illustrate the movement of blocks more clearly. These figures obviously show the performance of the DEM for modelling an assemblage of blocks under this type of loading.

Figure 5 shows the deformation of blocks under a shear wave loading. Induced relative horizontal displacements may be easily seen and the wave tends to shake the wall horizontally from left to right and from right to left. Because of the very high strength of the joints, there is no dislocation in the model.

Moreover, in the case of the combination of the two waves, figure 6 clearly shows the contribution of each waves. Here, both shear and compression waves have been given the same amplitude. The devastating action of this combination is characterized by two diagonal openings. This is a typical shear failure mode in which cracks or joints openings have usually a diagonal direction crossing the model with an angle of about 45° from the vertical direction.

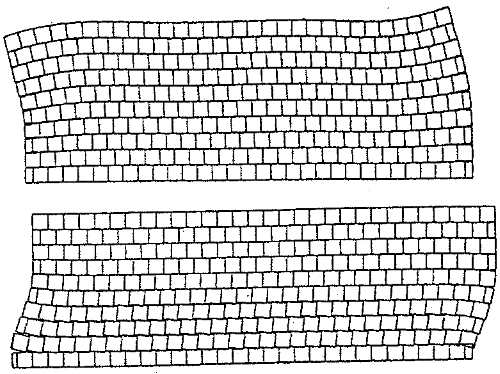

Figure 5. Effects of a shear wave only

The mechanical properties of the joints play a very important role regarding to the transmission of the waves through joints. To test the influence of these properties, a serie of calculations has been carried out using different values for the joint cohesion and the joint tensile strength. Moreover, the deformability of blocks has also been considered for this analysis. The loading was only a shear wave applied at the base of the model.

As shown on figure 7 (a) the absence of tensile strength at the joints (mortar bad adhered, masonry from category D) induces disorders in the masonry which is not moving anymore as a single entity. Diagonal openings also appear (see the right side of figure 7 (a)).

In addition to that, for the case of no cohesion joints, the joints do not resist to any shear stress. Consequently a sliding of the first row of blocks is taking place without affecting upper rows, figure 7 (b).

For the case where the joints have no tensile strength and no cohesion at the same time, figure 7 (c), the damages are more obvious than on previous models. The blocks are completely detached at the lateral sides of the model. A sliding has been observed between the first row of blocks and the upper part of model (see the vertical joints of the first row of blocks which became aligned). Diagonal open-

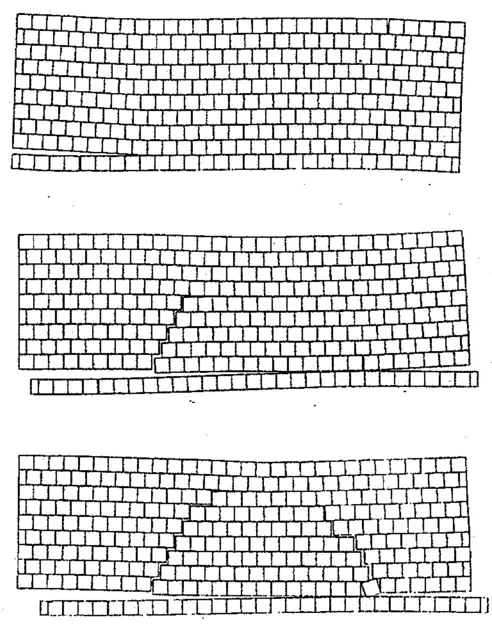

Figure 6. Effects of a combination of S- and P- waves

ings are also observed.

Finally, for the case where the joints have no tensile strength, no cohesion and where the stones have a Young modulus hundred times less than the last cases, the damages are observed in the form of the opening of vertical joints almost every where, especially at the lateral boundaries of the model where the joints have the maximum shear displacements figure 7 (d).

Parametric studies on dynamic discontinuous models carried out with UDEC, exhibit the complexity of such analysis. Many parameters have to be taken into consideration such as the mechanical characteristics of blocks and joints, the boundary conditions, the damping parameters, the time step, the amplitude, frequency, duration and type of dynamic loading, etc.

Case study 2 : a masonry side wall.

Earthquake or ground shaking causes dynamic movements in all three dimensions of the wall. In considering the side cross-section of the wall modelled so far, an attempt to find out the effects of the same shaking forces in another direction has been carried out.

The geometry of the model is very simple (figure 8) : 1 m width and 10 m height. The calculations are made under plane strain conditions. The model has the same mechanical properties as the previous models with joints having no tensile strength, no cohesion and the Young modulus of the stones being divided by 1000,

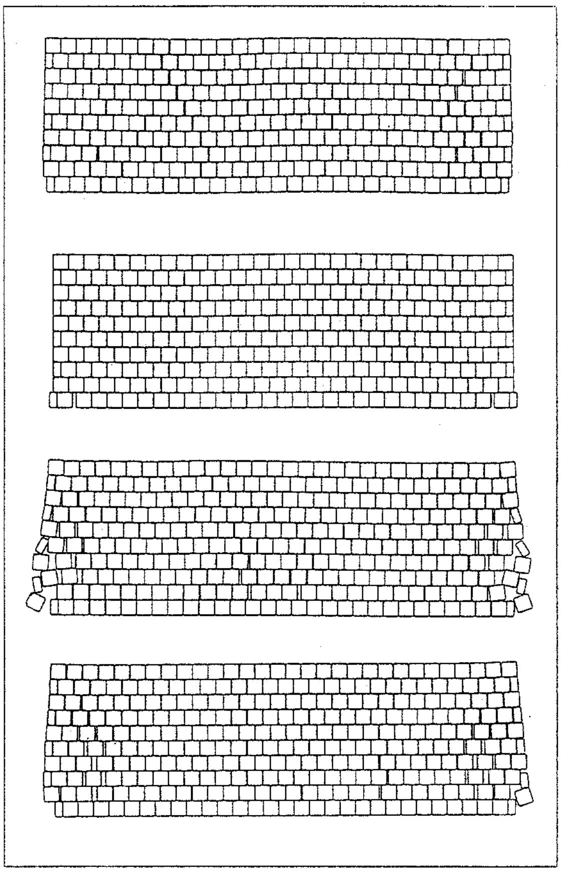

Figure 7. Response of the model for different mechanical properties of joints.

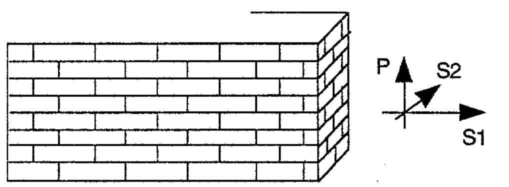

Figure 8. A wall subjected to 3 types of waves. In the case study 2, the effets of S2 only are studied

in comparison with the reference model.

The response of the model to the applied shear wave is illustrated on figure 9. The scenario of the loading begins with the movement of the base of the model towards the left.

It involves sliding because of the absence of joint cohesion. Consequently, the bottom of the model is becoming completely dislocated. Then the movement of the base driven to the right (after the change of the wave induced displacements) leads to the collapse of the upper blocks (as shown at the right side of figure 9).

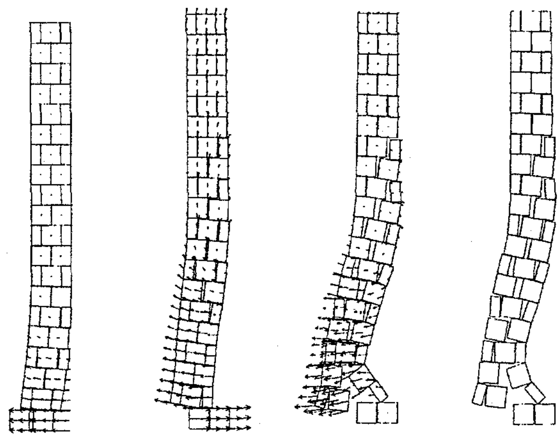

Figure 9. Scenario of failure of the side wall section, arrows indicate the amplitude and the direction of the velocity.

This mode of collapse is very familiar to the tall buildings (chemineys, columns, etc.), where the horizontal forces induce a horizontal displacement relatively large compared with the width of the structure. In that case, the gravity centre of the structure is not anymore located over the base and, consequently, the toppling of the superstructure occurs.

CONCLUDING REMARKS

On the basis of the results so far discussed, the following comments can be made :

- after using the DEM in modelling masonry structures, we found that this method may considerably improve investigations on the stability assessment of such structures. The concept of interacting deformable or rigid blocks separated by joints which may be given realistic deformation properties seems to be very well adapted to the study of ancient masonry structures (wall, arches, columns) as well as rock block constructions crossed by mechanical cracks or fissures (such as Memnon colossi in Egypt). The ability of the DEM and specially of UDEC to take into account the typical non-linear behaviour of such structures is of a very high importance in the analysis of ancient monuments. So, the DEM is supposed to answer to many engineers and researchers who always face this problem in using finite element codes ;
- the demonstrative examples showed how the responses of the model can differ according to many factors such as the type of the seismic waves and the mechanical characteristics of the mortars or stones. The obtained results have emphasized also the importance of the mechanical properties of the mortar as a significant factor determining the mode of failure of masonry structures.

- the shear failure modes of the wall (diagonal openings of joints or diagonal cracks) are obviously observed for the walls built with mortars having no tension strength and also for those having no cohesion (very degraded mortars). This is a typical case encountered in many old masonry buildings.

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